

Final exam Medical Physics

Wednesday, January 25th, 2017; 14:00 – 17:00

Clearly write on each answer sheet your name and student number.

Total: 90 points

Exam grade = (points/10) + 1

The total grade for this course will be calculated as:

Final grade = $(2 \times (\text{average grade from 3 tests}) + 4 \times (\text{presentation grade}) + 4 \times (\text{exam grade})) / 10$

1) Radiation physics (10 pt)

- a) The half-life of a radioactive isotope
- is the time it takes to disappear the radioactive isotope completely.
 - is the time it takes for half of the nuclei in a sample of the isotope to decay.
 - is needed to split a nucleus of an atom into half. (2 pt)
- b) Compton scattering occurs on
- the atomic nucleus.
 - strongly bound electrons of an atom.
 - free electrons of an atom. (2 pt)
- c) Primary ways of transferring energy to matter by a charged particle are
- annihilation.
 - emission of an Auger electron.
 - ionization and excitation. (2 pt)
- d) Spontaneous radioactivity takes place when
- the incoming particle has bigger energy than the target atom.
 - the incoming particle has smaller energy than the target atom.
 - the nucleus decays by itself under emission of radiation. (2 pt)
- d) Heavy charged particles
- scatter a lot due to their mass.
 - are charged particles with a mass bigger than that of the electron.
 - only excite atoms. (2 pt)

2) Biological effects and molecular mechanisms of ionizing radiation (10 pt)

- a) Order the following 3 types of radiation with respect to their associated LET from low (1) to high (3).
- | | |
|---|---|
| <input type="checkbox"/> 20 keV X-rays | 2 |
| <input type="checkbox"/> 1 GeV Carbon ions | 3 |
| <input type="checkbox"/> 60 keV X-rays (2 pt) | 1 |
- b) Indirect radiation damage refers to
- Cell death occurring a long time after exposure.
 - Cell death after unsuccessful DNA repair.
 - DNA damage not induced by the primary radiation but e.g. by OH radicals. (2 pt)

- c) Plasmid DNA is a standard system for studies on biological radiation damage.
- Plasmid DNA exhibits a dramatic conformational change upon SSB or DSB that is easily detected.
 - Plasmid DNA is single stranded.
 - Plasmid DNA is more sensitive to radiation than human DNA. (2 pt)
- d) Which of the 4 types of radiation is best suited for targeted irradiation of a solid tumor?
- Heavy ions
 - Electrons
 - X-ray photons
 - Neutrons (2 pt)
- e) ^{131}I can be administered to a person where it accumulated in the thyroid. α -rays and β -particles are emitted during the decay and the γ -rays can be detected for imaging the organ. Clearly, the dominating decay process must involve the emission of a γ -photon. What other factors have to be considered?
- The β -particles cannot be detected, but deposit dose in the organ, increasing cancer risk.
 - The β -particles only deposit dose in the surrounding tissue.
 - The β -particles escape the body and can damage the electronics of the imaging device. (2 pt)

3) Optics in modern medicine (15 pt)

- a) At an interface between an unknown transparent material and water, a ray of light is incident from the water side at an angle of 30° to the normal. The refracted ray makes an angle of 25° with the normal. What is the index of refraction for the unknown material? (5 pt)

2-2) For this problem, the incident angle, $\theta_1 = 30.0^\circ$, and the refracted angle is $\theta_2 = 25.0^\circ$. The index of refraction of water is $n_1 = 1.33$ (Table 2-1). We can use Snell's Law to determine the index of refraction of the unknown medium:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2} = \frac{1.33 \times \sin 30.0^\circ}{\sin 25.0^\circ} = 1.57$$

The index of refraction of the unknown material is 1.57. This makes sense because the angle of refraction is smaller when a ray travels from a medium of low to one of high index of refraction.

- b) What is the critical angle for the interface described in a)? Make a sketch of the interface including the geometry of rays that would undergo total internal reflection. (5 pt)

$$\theta_{\text{crit}} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.33}{1.57}\right) = 57.9^\circ$$

Since total internal reflection can take place only when rays travel from a high index material into a low index material, we have chosen the greater of the two indices as n_1 .

c) Compute the power densities for photocoagulation of the retina using 500 μm diameter spots and a laser power of 200 mW and for photovaporization of opacities within the eye using 50 μm diameter spots and a laser power of 5 W. What is the ratio between these power densities and what is the physical reason for this ratio? (5 pt)

P3.3) Typical power densities for ophthalmological surgery applications can be obtained by dividing the intensity of the laser by the area of the focused spot (Eq. 3.7):

$$I = \frac{P_{laser}}{A}$$

For part a) the relevant values were a 500 micron spot diameter and a power of 200 milliwatts for photocoagulation of the retina and other parts of the eye. The area of a circular spot is πr^2 , where r is the radius (one-half of the diameter) and $\pi = 3.14159$. This corresponds to a power density of:

$$\begin{aligned} I &= \frac{P_{laser}}{A} \\ &= \frac{200 \times 10^{-3} \text{ watts}}{\pi \left(\frac{500 \text{ micron}}{2} \times \frac{1 \text{ cm}}{10^4 \text{ micron}} \right)^2} \\ &= 100 \frac{\text{watts}}{\text{cm}^2} \end{aligned}$$

b) Photovaporization of cataracts or opacities within the eye might use a 50 micron-diameter spot and a power/intensity settings of roughly 2 watts, corresponding to a power density of:

$$I = \frac{2 \text{ watts}}{\pi \left(\frac{50 \text{ microns}}{2} \times \frac{1 \text{ cm}}{10^4 \text{ microns}} \right)^2} = 10^5 \frac{\text{watts}}{\text{cm}^2}$$

c) Comparing the two settings by dividing our answers for the power densities in parts a) and b), we see that the photovaporization configuration has a power density *1000 times higher* than that for photocoagulation. That extra intensity allows surgeons to vaporize the tissue, rather than merely slowly heating it for purposes of photocoagulation.

4) X-ray absorption (15 pt)

some linear attenuation coefficients:

X-ray energy	$\mu_{air} (\text{cm}^{-1})$	$\mu_{water} (\text{cm}^{-1})$	$\mu_{fat} (\text{cm}^{-1})$	$\mu_{bone} (\text{cm}^{-1})$
20 keV	6.4×10^{-4}	0.76	0.5	4.8
60 keV	3.7×10^{-5}	0.20	0,17	0.47

a) What percentage of the incident X-rays are transmitted by a 1-cm-thick rib embedded in 20 cm of soft tissue for 20 and 60 keV X-rays? (Assume, that for 20 cm of soft tissue, the transmission is 2.1×10^{-7} for 20 keV and 1.5×10^{-2} for 60 keV). (5 pt)

- b) What percentage of the incident X-rays are transmitted by a 4 cm region of breast tissue for 20 and 60 keV X-rays? (Approximate the soft tissues in both parts as having the same linear attenuation coefficient as water). (5 pt)
- c) Why is lead used for shielding X-rays? (5 pt)

P5-3) a) For the case of the 1 cm thick rib, we have $x = 1$ cm and for 20 keV $\mu_{\text{bone}} = 4.8$ cm^{-1}

$$\begin{aligned} \text{Percent transmission} &= 100\% \times e^{-\mu x} \\ &= 100\% \times e^{-4.8 \text{cm}^{-1} \times 1 \text{cm}} \\ &= 0.82\% \end{aligned}$$

Note that if we always use the lower value for For 60 keV, $\mu_{\text{bone}} = 0.55$ cm^{-1}

$$\begin{aligned} \text{Percent transmission} &= 100\% \times e^{-0.55 \text{cm}^{-1} \times 1 \text{cm}} \\ &= 58\% \end{aligned}$$

From the text, we learned that 2.1×10^{-7} of the original beam is transmitted by 20 cm of soft tissue alone for 20 keV x-rays; for 60 keV x-rays, 1.5×10^{-2} of the original x-rays are transmitted. We multiply this factor by the two results above to get a total transmission at 20 keV of $0.82\% \times 2.1 \times 10^{-7} = 1.7 \times 10^{-7}\%$ and at 60 keV of 0.87% .

(b) For the 4 cm region of breast tissue, we have transmissions of:

$$20 \text{ keV: } \frac{I_{\text{trans}}}{I_o} = e^{-\mu x} = e^{-0.76 \text{cm}^{-1} \times 4 \text{cm}} = 0.048$$

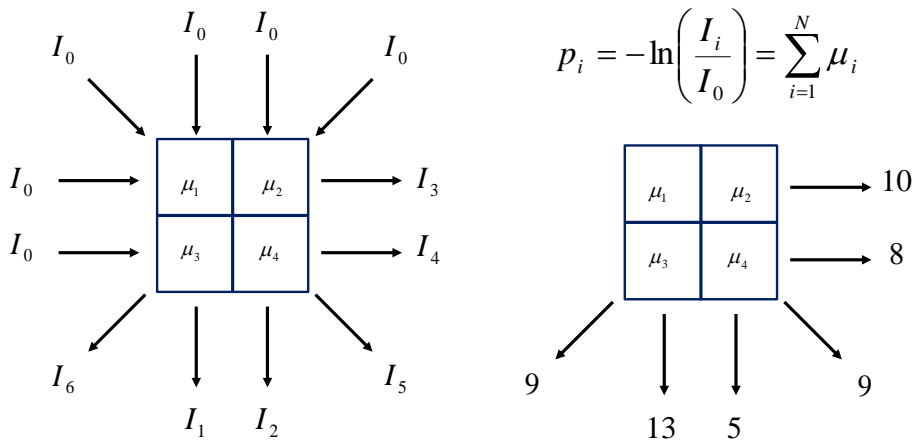
$$60 \text{ keV: } \frac{I_{\text{trans}}}{I_o} = e^{-\mu x} = e^{-0.20 \text{cm}^{-1} \times 4 \text{cm}} = 0.43$$

As pointed out in the text, more higher energy x-ray photons are transmitted through breast tissue to develop the image receptor. However, as the following problem shows, raising the energy degrades the contrast.

c) very high atomic number

5) X-ray and computed tomography (15 pt)

A 2D object consisting of four equally sized pixels is imaged by x-rays (left figure). The linear attenuation coefficients of the pixels are $\mu_i, i = 1 \dots 4$. The different projections p_i that are measured are given in the figure on the right.



- a) Find the exact values $\mu_i, i = 1 \dots 4$ using iterative reconstruction. Show all iterations. (8 pt)
- b) Estimate the values $\mu_i, i = 1 \dots 4$ using back projection. (7 pt)

6) Radiation therapy (15 pt)

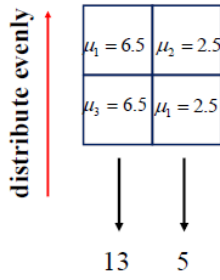
- a) Why are X-ray photons and electrons the most commonly used particles for cancer therapy? Give at least two reasons. Explain the advantage of using protons as a particle for radiation therapy. (5 pt)

Relatively easy to produce, relatively cheap, high penetration depth allows for irradiation of essentially any tumour. Protons are stopped at an energy dependent depth. Dose is maximum just before the end of the track (Bragg peak). Tumour can be targeted.

- b) The naturally occurring potassium-40 accounts in the human body decays by emission of beta particles and gamma photons. Assume, that only the beta particles are absorbed in the body and contribute to the radiation dose to the body. Each decay deposits on average 0.39 MeV. The total activity of potassium-40 in the body is 4630 Bq. What is the annual radiation dose to the body in Gy, assuming a body weight of 70 kg. How does the result compare to other sources of background radiation? (10 pt)

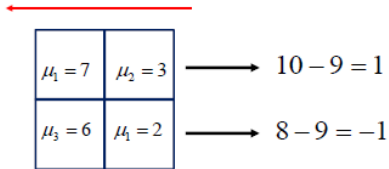
a)

Step 1

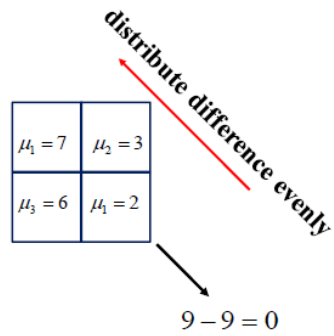


Step 2

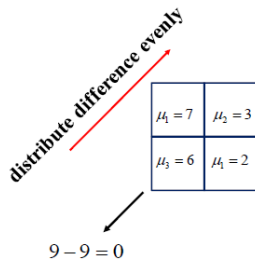
distribute difference evenly



Step 3

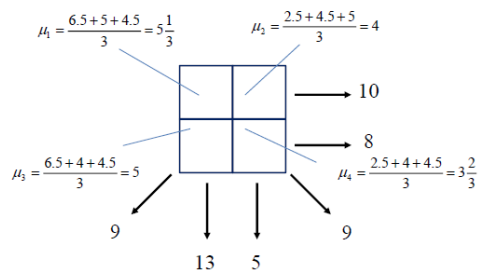


Step 4



b)

All projections have to be evenly projected back and weighted by the number of projections through the pixel. This number is three for every pixel. There is only one step.



P7-2) The source activity of potassium-40 has been provided per kg of body weight as:

$$4630 \text{ Bq}/70 \text{ kg} = 66 \text{ Bq}/\text{kg}.$$

The half-life of the isotope is not relevant in this case, as the body's supply is constantly being refreshed from the environment, so we can treat this as a constant source activity. For an annual dose, we are interested in an exposure time, T, of 1 year or:

$$\begin{aligned} T &= 1 \text{ year} \times 365 \text{ days/year} \times 24 \text{ hours/day} \times 60 \text{ min/hour} \times 60 \text{ sec/min} \\ &= 3.15 \times 10^7 \text{ sec} \end{aligned}$$

The energy (in Joules) released in every radioactive decay is given by the energy of the beta particles, 0.39 MeV, where $1 \text{ MeV} = 10^6 \text{ eV}$. However, we need to express the energy in Joules. This can be accomplished using the conversion factor:

$$\begin{aligned} E(\text{Joules}) &= E(\text{eV}) \times \left(\frac{1.60 \times 10^{-19} \text{ Joules}}{\text{eV}} \right) \\ &= 0.39 \text{ MeV} \times 10^6 \frac{\text{eV}}{\text{MeV}} \times 1.6 \times 10^{-19} \frac{\text{Joule}}{\text{eV}} \\ &= 6.2 \times 10^{-14} \text{ Joules} \end{aligned}$$

The dose is then:

$$\begin{aligned} \text{Dose} &= 66 \text{ Bq}/\text{kg} \times 1 \text{ decay/sec/Bq} \times 6.2 \times 10^{-14} \text{ Joules} \times 3.15 \times 10^7 \text{ sec} \\ &= 0.00013 \text{ Joule}/\text{kg} = 0.00013 \text{ Gy} = 0.13 \text{ mGy} \end{aligned}$$

This value (0.13 mGy) constitutes about one-third of the total annual dose from all internally deposited radioisotopes.

7) Molecular Imaging – questions (10 pt)

Note: Each question has one and only one correct answer.

- a) Which of the following statements related to the tracer principle is **not correct**:
- A molecule labeled with a radioactive isotope (radiotracer) behaves in the same way as the non-labeled molecule.
 - Radiotracers are used for both diagnostic imaging and radiotherapy.
 - The behaviour of a molecule labeled with a radioactive isotope depends on whether the radioactive isotope decays via beta+ or beta– decay. (2 pt)
- b) A radioisotope can often be attached to different molecules, thus forming different radiotracers. For a particular radioisotope, the radiation dose to the patient
- is highest for the radiotracer with the longest biological half-life.
 - is highest for the radiotracer with the shortest biological half-life.
 - does not depend on the biological half-life. (2 pt)
- c) The spatial resolution of a gamma camera
- is mainly determined by the detector.
 - is mainly determined by the collimator.

- is, depending on the design of the gamma camera, mainly determined by either the detector or the collimator. (2 pt)
- d) Which one of the following statements is **not correct** ?
- The ratio of scattered to true coincidences is independent of the amount of radiotracer injected into the patient.
 - The random coincidence rate is smaller for a smaller coincidence time window.
 - The random coincidence rate scales linearly with the amount of radiotracer injected into the patient. (2 pt)
- e) Consider a time-of-flight (TOF) PET scanner with a coincidence resolving time (CRT) of 500 picoseconds. What is the spatial resolution Δx along the line-of-response that follows from this CRT ?
- 7.5 cm
 - 15 cm
 - 30 cm (2 pt)